Towards High Accuracy Robot-Assisted Surgery ⋆

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Abstract: In this article, we propose a new error modeling approach for robot manipulators in order to improve the absolute accuracy of a tools pose by using polynomial regression method. The core idea is based on a well-known fact: accuracy of repeatedly reaching a given position is much higher than the accuracy of absolute positioning (i.e. moving the manipulator to a given position). The underlying reason is, that positioning errors are dominated by systematic errors - while stochastic errors are significantly smaller. This fact is then exploited to apply some learning algorithm to derive an error-compensation model. Technically, this means that the robot is being calibrated a priori with some external sensor once. Afterwards it can operate with a much better quality. In detail, we propose to first perform a coordinate transformation using a least mean square approach (for registration). Then, to account for deviations of measured position in comparison to nominal (robot) position, the frame transformation model at each robots joint is extended by translational and rotational error parameters. This is then used to built an error compensation model with regression techniques. We evaluate the method on a data set obtained using a 7DOF robot manipulator and show that this approach brings positioning error to the order of repeatability errors for this manipulator.

Keywords: Robots manipulators, intelligent robotics, modeling, error quantification.

1. INTRODUCTION

In many medical and industrial applications, a high absolute pose accuracy of the robot's end effector is a crucial requirement (Mavroidis et al. (1997)). Even small errors in robot's tool center point (TCP) pose (position and orientation) can cause dangerous effects. For instance, a recent clinical study (Lonjon et al. (2016)) in spinal surgery confirmed, that robot-assisted screw placement accuracy was not satisfactory to significantly reduce complications.

Robots generally have higher repeatability then the absolute accuracy. As long as repeatability is low the absolute accuracy can be improved with calibration. An overview of different calibration techniques can be found in Mooring et al. (1991) or Karan and Vukobratović (1994). However, for the frequently changing tasks in medical applications most such methods are very impractical.

The main sources for positioning inaccuracies can be divided into systematic (or geometric) and stochastic (or non-geometric) errors. *Geometric errors* are present when nominal kinematic parameters of the robot manipulator don't correspond to actual kinematic parameters. One of the widely used approaches to model these errors consists of extending the four parameters of the Denavit-Hartenberg (D-H) model with errors as in Veitschegger and Wu (1986). However, if the robot has consecutive revolute joints with near parallel axes this notation can lead to convergence problems during parameter identification. Methods that address this problem includes a modification of the D-H approach. For instance, Hayati and Mirmirani (1985) introduces an extra rotation about an axis orthogonal to the plain in which these parallel axes lie. Methods not relying on D-H formalism include Zero-Reference (Mooring (1983)), and Product-of-Exponentials (POE) models (Okamura and Park (1996)).

Non-geometric errors are attributed to transmission non-linearities, elastic deformations and thermal expansion. To account for these errors corresponding effects should be analytically expressed with gear or elastic models as in Duelen and Schröer (1991) or Judd and Knasinski (1990). In many applications it suffices to restrict to geometric errors to simplify calibration process, but it is not desirable if high accuracy is required. Obtaining an accurate error model and, consequently, a corrected kinematic model has been extensively studied in the last two decades. Recent research adds non-geometric parameters to selected kinematic model of the manipulator, increasing the number of unknown parameters to be identified as in Marie et al. (2013) or Nubiola and Bonev (2013). Considering all sources that contribute to end-effector pose errors, it is difficult to model all relevant parameters. As a result, global calibration that explicitly models error sources and thus covers the whole workspace of the robot is difficult to achieve. As an alternative, we propose a local calibration with measurements taken in the area of interest and subsequent building an error model that does not attribute errors to specific physical processes, but distributes

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geometric and non-geometric influences over introduced error parameters. Robotic tasks in medical assistance are typically unique (i.e. each patient is different), and robot-human interaction is very often required. Hand guided robots are used which are designed for such applications (e.g., KUKA's LBR IIWA). However, such robot flexibility introduces additional (systematic) errors which decrease positioning accuracy a lot.

We propose an error modeling method for a robot manipulators in order to account for geometric and non-geometric errors to improve the positional and orientational accuracy of a robot's tool. In Section 2, an estimation method for translational and rotational errors is proposed based on reference data of the robot's flange measured by a tracking system and corresponding nominal poses retrieved from a robot control. Using regression analysis (Section 3), we model these errors as functions of the robot's state (including joint configurations and pose of the TCP) that was interpolated from reference points. Consequently, the errors, which are represented in the given data set, can be distributed over defined error parameters. In Section 4, we applied this method to the performance evaluation of the KUKA LBR IIWA manipulator. In our experiment, we reduced the robot's absolute errors to the order of repeatability errors.

2. ERROR PARAMETERS FOR ROBOT MANIPULATORS

In this section, we describe a method used for the generalized error parameter modeling. Given a sequence of reference poses of a robot flange $\mathbf{P}_T \in \mathbf{SE}(3)$, measured by a tracking system along with corresponding nominal poses of a robot flange $\mathbf{P}_R \in \mathbf{SE}(3)$, and robot joint values $\theta_i, i \in [1, ..., n]$ (where n is number of robot joints) retrieved from a robot control. The goal is to find an error parameter model that accounts for geometrical as well as non-geometrical errors distributed over the nominal poses in order to minimize the absolute difference to the reference poses. For that, first, D-H representation and generalized error modeling is used to establish the transformation from the base frame of the robot to the flange. Then, for every reference-nominal pose pair, we calculate the error parameters. Finally, using regression analysis (in Section 3), we determine the coefficients of the error model that optimally fits given input pose pairs.

2.1 D-H representation

D-H parameters are used to describe the kinematics of a manipulator. The position and orientation of each frame A_i with respect to the previous reference frame A_{i-1} is defined by a homogeneous transformation, that depends on the geometric parameters of the manipulator. Namely, skew angles α_i, link lengths a_i, joint offsets d_i and joint angle offsets θ_i (in this and subsequent sections we use $c_\alpha := cos(\alpha)$, $s_\alpha := sin(\alpha)$ for compact notation):

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

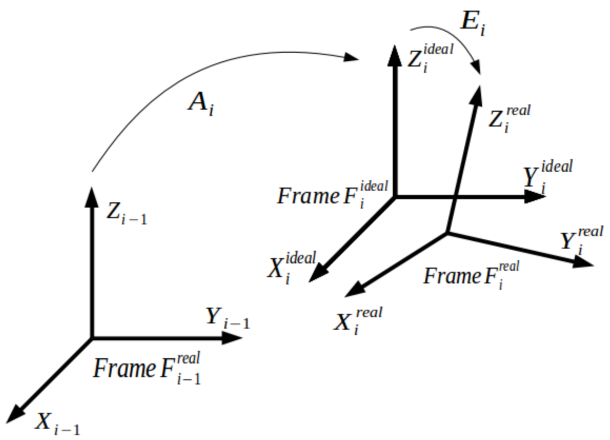

Fig. 1. Frame transformations in presence of errors

The transformation A_T from the base to the end-effector of a robot with n joints is then obtained by:

$$A_T = \prod_i^n A_i. \quad (2)$$

2.2 Generalized error parameters

To account for difference between the nominal pose (position and orientation) $\mathbf{p}_{iR} \in \mathbf{P}_R$ of the end-effector, provided by the robot, and the corresponding reference pose $\mathbf{p}_{iT} \in \mathbf{P}_T$, measured by an external tracking system, the kinematic robot model (represented by equation (2)) can be extended by translational and rotational error parameters. The introduction of these errors leads to displacement of joint frames from their nominal locations (Fig. 1). For joint i this difference in frames can be represented by a homogeneous matrix E_i with 6 error parameters called generalized error parameters: $e = (e_{i1}, \ldots, e_{i6})$. The rotational part of matrix E_i consists of e_{i4}, e_{i5}, e_{i6}, which denote rotation about X,Y and Z axes with respect to A_i. The e_{i1}, e_{i2}, e_{i3} represent translation in X,Y and Z direction respectively.

Therefore, a generalized error for a frame A_i can be expressed as:

$$\boldsymbol{E}_i = T(e_{i1}, e_{i2}, e_{i3}) R(x_i, e_{i4}) R(y_i, e_{i5}) R(z_i, e_{i6}), \quad (3)$$

where $R(\ldots)$ stands for the rotational and $T(\ldots)$ for the translational errors.

Multiplying mentioned transformations leads to a common error matrix for frame A_i:

$$\boldsymbol{E}_i = \begin{bmatrix} c_{e5} c_{e6} & c_{e6} s_{e4} s_{e5} - c_{e4} s_{e6} & c_{e4} c_{e6} s_{e5} + s_{e4} s_{e6} & e_1 \\ c_{e5} s_{e6} & c_{e4} c_{e6} + s_{e4} s_{e5} s_{e6} & -c_{e6} s_{e4} + c_{e4} s_{e5} s_{e6} & e_2 \\ -s_{e5} & c_{e5} s_{e4} & c_{e4} c_{e5} & e_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Assuming that the errors are expected to be small, (4) can be simplified by applying Taylor's expansion and first order approximation to trigonometric functions and products. With this linearization, where $s_\alpha = \alpha$, $c_\alpha = 1$ and $s_\alpha s_\beta = 0$, the generalized error model E_i for frame A_i becomes:

$$
\boldsymbol{E}_i = \begin{bmatrix} 1 & -e_{i6} & e_{i5} & e_{i1} \\ e_{i6} & 1 & -e_{i4} & e_{i2} \\ -e_{i5} & e_{i4} & 1 & e_{i3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

Considering the introduced error parameters, the general transformation model A_E in presence of translational and rotational errors for a robot with n joints, is given by:

$$
A_E = \prod_{i}^{n} A_i E_i \tag{6}
$$

Often, the rotational parameters are more error-prone comparing to translational errors. We decided to examine three different error models:

(1) *RotZ*-error model resulting in n error parameters:

$$
\boldsymbol{E}_i = \begin{bmatrix} 1 & -e_{i6} & 0 & 0 \\ e_{i6} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}
$$

(2) *RotXYZ*-error model resulting in $3 \times n$ error parameters:

$$
\boldsymbol{E}_i = \begin{bmatrix} 1 & -e_{i6} & e_{i5} & 0 \\ e_{i6} & 1 & -e_{i4} & 0 \\ -e_{i5} & e_{i4} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}
$$

(3) *RotXYZTransXYZ*-error model as in equation (5), leading to $6 \times n$ error parameters.

2.3 Error parameters calculation

The nominal pose of the end-effector defined by (6) should be as close as possible to the measured reference pose $\mathbf{p}_{iT} \in \mathbf{P}_T$. It can be written in general form as:

$$
\mathbf{p}_{iT} = \boldsymbol{f}_e(\boldsymbol{e}), \tag{9}
$$

where $\boldsymbol{f}_e$ is a non-linear function of error parameter vector $\boldsymbol{e}$. Since these errors are small we can linearize this function at $\mathbf{0}$ (all error parameters are equal to 0):

$$
\mathbf{p}_{iT} = \boldsymbol{f}_e(\mathbf{0}) + \boldsymbol{J}_e|_{\mathbf{0}}(\boldsymbol{e} - \mathbf{0}), \tag{10}
$$

where $\boldsymbol{J}_e|_{\mathbf{0}}$ is a Jacobian function of $\boldsymbol{f}_e$ with respect to the elements of error vector $\boldsymbol{e}$, evaluated at $\mathbf{0}$ (denoted as $\boldsymbol{J}_e$ in the following):

$$
\boldsymbol{J}_e = \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{e}} \tag{11}
$$

It should be noted that $\boldsymbol{f}_e(\mathbf{0})$ corresponds to nominal pose $\mathbf{p}_{iR} \in \mathbf{P}_R$, given by A_T. If the difference between nominal position and measured position, $\boldsymbol{\Delta}\mathbf{p}_i$ is

$$
\boldsymbol{\Delta}\mathbf{p}_i = \mathbf{p}_{iT} - \mathbf{p}_{iR}, \tag{12}
$$

then substituting (10) in (12) we get

$$
\boldsymbol{\Delta}\mathbf{p}_i = \boldsymbol{J}_e \boldsymbol{e} \tag{13}
$$

which can be solved with a least mean square approach for every measured-nominal point pair.

2.4 Indistinguishable Error Parameters

Some generalized errors from link $i-1$ contribute to the same end-effector pose errors as errors from link i (Meggiolaro and Dubowsky (2000)). Since the effects of such error parameters can not be distinguished, the contributions are divided equally, due to the least mean square approach used for their calculation. For each link i following combinations are always present:

$$
\begin{aligned}
e_{i2} &= e_{i-13} s_{\alpha_i} = e_{i-16} a_i c_{\alpha_i} \\
e_{i3} &= e_{i-13} c_{\alpha_i} = -e_{i-16} a_i s_{\alpha_i} \\
e_{i5} &= e_{i-16} s_{\alpha_i} \\
e_{i6} &= e_{i-16} c_{\alpha_i}.
\end{aligned} \tag{14}
$$

If joint i is prismatic additional combinations are present:

$$
\begin{aligned}
e_{i1} &= e_{i-11} \\
e_{i2} &= e_{i-12} c_{\alpha_i} = e_{i-13} s_{\alpha_i} = e_{i-16} a_i c_{\alpha_i} \\
e_{i3} &= -e_{i-12} s_{\alpha_i} = e_{i-13} c_{\alpha_i} = -e_{i-16} a_i s_{\alpha_i} \\
e_{i5} &= e_{i-16} s_{\alpha_i} \\
e_{i6} &= e_{i-16} c_{\alpha_i}.
\end{aligned} \tag{15}
$$

When only positional part of the end-effector pose is considered and the last joint n is revolute and $a_n = 0$:

$$
\begin{aligned}
e_{n-11} &= e_{n-15} d_n \\
e_{n-12} &= -e_{n-14} d_n.
\end{aligned} \tag{16}
$$

3. REGRESSION ANALYSIS

After calculating the values of the error parameters for every point pair and eliminating the redundant parameters, an estimator based on the obtained data can be constructed. For every introduced error parameter regression analysis can be used to predict its value depending on input variables. These variables, often called independent variables and are denoted by $\boldsymbol{X}$. The unknown parameters that define the model are estimated from the data (Rawlings et al. (2001)).

3.1 Modeling of error parameters

One of the basic forms of regression analysis is linear regression. It uses Least Square Method to find a model that fits the observed data by minimizing the sum of the squared deviations between the observation and estimator:

$$
min||\boldsymbol{X}\boldsymbol{\omega} - y||_2^2, \tag{17}
$$

where y represents the target variable, and ω regression coefficients. In order to model nonlinear relationships input variables can be extended by :

$$
\boldsymbol{X} = [x_1, x_1^2 \dots x_1^p \dots x_n^p] \tag{18}
$$

where p is the degree of polynomial. By considering these input parameters as independent, linear modeling can be applied.

While a higher degree polynomial can better fit the observed data, this can result in over-fitting and poor performance on new data. Ridge Regression (Hoerl and Kennard (1970)) is a technique that uses regularization by minimizing a penalized residual sum of squares (19). This approach is effective in case of highly correlated input variables, like the ones resulting from polynomial expansion.

$$
min||\boldsymbol{X}\boldsymbol{\omega} - y||_2^2 + \alpha||\omega||_2^2, \tag{19}
$$

where $\alpha \geq 0$ is the shrinkage parameter that controls the amount of regularization: the larger the value of α, the greater the amount of shrinkage and thus the coefficients become more robust to collinearity.

3.2 Model training

We split the given data set into training (50%), validation (25%) and testing (25%) sets. The training set is used to determine the regression coefficients of the model, validation set is used for tuning model parameter α and selecting the best model, while the performance evaluation is done on the test set that was not used in any steps of modeling. As input parameters, the joint configuration of the manipulator and the nominal position of the end-effector were considered. To reduce the number of possible combinations of these input parameters and thus the number of models to select from, the correlation coefficient between all available input parameters and output variables is calculated. Only those parameters that displayed significant correlation were used for modeling.

To evaluate the performance of regression models, R^2 metric (the coefficient of determination) is used. It provides a measure of how well test data are likely to be predicted by the model. The best score corresponds to 1.0. If $\hat{y}_i$ is the predicted value and $\bar{y}$ is the mean, then:

$$R^2(y, \hat{y}) = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}. \tag{20}$$

4. EVALUATION

For evaluation of our method, we used the data sets measured by Siemens Healthcare GmbH.

4.1 Experimental setup

Experimental setup consists of LBR IIWA R800 and a Faro laser tracking system that was used to measure the position of a reflector, rigidly mounted on the robot flange. The accuracy of the laser tracker is up to $0.015mm$. First part of the dataset contains 2335 points (Cartesian position) in measuring volume of $240 \times 680 \times 640mm$, measured by the tracker, and nominal position as well as configuration of the joints provided by the robot. The second part includes data for 875 points in $170 \times 520 \times 520mm$ measuring volume with Cartesian position from three reference points fixed on the flange and corresponding nominal data. The orientation of the robot's flange was computed from these three points, provided by the laser tracker. The data from the tracker was transformed into the coordinate system of the robot using a least mean square and Singular Value Decomposition approach as in (Lorusso et al. (1995)). The D-H parameters of LBR IIWA R800 are obtained from nominal data according to the official KUKA's specification.

If we consider the case of full pose measurement, differences in position and orientation can not be compensated in the best way, because they are not properly weighted. Considering that there is no physical meaning attached to error parameters we can divide them in two parts and thus overcome this difficulty. To do so, we find error parameters that only influence position of the TCP and those that only have effect on its orientation and attribute to them positional and rotational differences respectively. The evaluation results of the positional part are presented for models obtained from the data set containing only positional data because they are analogues to results from the positional part of the full pose data set.

4.2 Positional model

For the case when only the position of the end-effector is measured, $\mathbf{\Delta p}_i$ is a vector:

$$\mathbf{\Delta p}_i = [x_{iT} - x_{iR}, y_{iT} - y_{iR}, z_{iT} - z_{iR}]^T \tag{21}$$

Considering that LBR IIWA R800 has all revolute joints, (14) and (16) lead to the following equivalences:

$$\begin{gathered} e_{22} = e_{13} \quad e_{25} = e_{16} \quad e_{32} = e_{23} \quad e_{35} = e_{26} \\ e_{42} = -e_{33} \quad e_{45} = -e_{36} \quad e_{52} = -e_{43} \quad e_{55} = -e_{46} \\ e_{56} = e_{65} = e_{61}d_7 \quad e_{62} = e_{53} \\ e_{64} = -e_{62}d_7 \quad e_{73} = e_{63} \end{gathered} \tag{22}$$

When constructing the $\boldsymbol{J}_e$, it should be noted that not only some error parameters do not influence the position of the end-effector but its general form differs for different error models.

RotZ-error model In case of errors about Z axis error parameters e_{66} and e_{76} have no effect on position, resulting in 5 unknown parameters leading to $\boldsymbol{J}_e$ as a 3×5 function of vector $\boldsymbol{e} = [e_{16}, e_{26}, e_{36}, e_{46}, e_{56}]^T$. Using equation (13), we find values of considered errors for every data point in training and validation set. Then, the input parameters were chosen from potential candidates which are Cartesian position of the TCP (X, Y, Z) and first six joint angles $(\theta_1, \ldots, \theta_6)$, because θ_7 has no effect on the position of the TCP. For example the correlation between input parameters and calculated value of e_{16} ranges from 0.9295 for θ_4 to 0.0660 for θ_1. As a result θ_4, and others that showed greater correlation were chosen as potential input parameters for the e_{16}.

For the selected model candidates the best hyper parameter for ridge regression were chosen by training the model with training set and choosing parameter corresponding to the highest R^2 value for the validation set. To choose the final configuration of input parameters the progression of R^2 value for the validation set was analyzed. Namely how an addition of another input parameter or increase of polynomial degree influenced the value of R^2. After analyzing the resulting models on the test set we could observe that the coefficient of determination for e_{46} (0.5918) was smaller than of the other error parameters (ranging from 0.8524 to 0.9167), although it still reflects the general tendencies of the parameter.

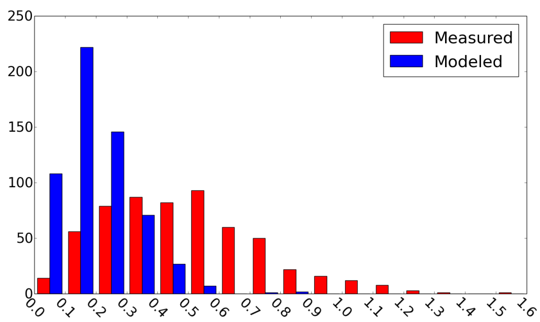

Fig. 2. Distance error (mm) frequency plot of *RotZ*-error positional model

To illustrate the results of the modeling, error parameters were applied to the test set and the corrected positions of the end-effector were calculated. Fig. 2 presents the frequency plot of positional errors, measured as the distance between the nominal and measured position (red) alongside with the distance from modeled to measured position (blue). It shows a significant shift of error distribution towards zero.

RotXYZ-error model Considering all rotational errors, the unidentifiable parameters are e_{66}, e_{74}, e_{75}, e_{76} and $\boldsymbol{J}_e$ is a 3×17 function of vector $\boldsymbol{e} = [e_{14}, e_{15}, e_{16}, \dots, e_{64}, e_{65}]^T$.

As previously observed in *RotZ*-error model one of the error parameters had R^2 value smaller compared to others. In both models this is mainly caused by present effects that can not be fully modeled by used error parameters.

Even with these effects, the distance between nominal and measured position and the distance from modeled to measured position is significantly decreased as shown by frequency plot in Fig. 3. It also shows further decrease of distribution's width compared to *RotZ*-error model.

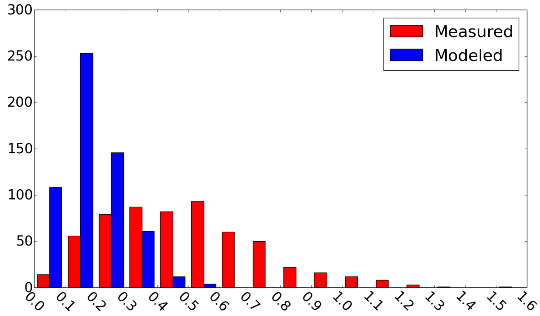

Fig. 3. Distance error (mm) frequency plot of *RotXYZ*-error positional model

RotXYZTransXYZ-error model For the most general case of all possible errors the unidentifiable parameters are the same as for the *RotXYZ*-error model, and $\boldsymbol{J}_e$ is a 3×38 function of vector $\boldsymbol{e} = [e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, \dots, e_{73}]^T$.

As expected, the R^2 values of the selected models were comparably high for all error parameters, because this model can represent all possible errors. The results of modeling are presented in Fig. 4 which show results similar to *RotXYZ*-error model.

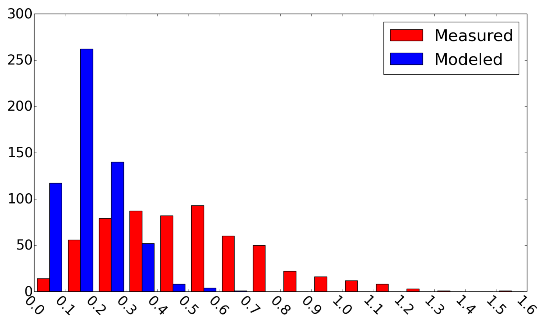

Fig. 4. Distance error (mm) frequency plot of *RotXYZ-TransXYZ*-error positional model

4.3 Orientational model

We first need to establish representation of the rotational part of the pose. While it consists of a 3 × 3 matrix, it only has 3 degrees of freedom. Here we consider parametrization by a vector $\boldsymbol{w} = [w_1, w_2, w_3]^T$ representing rotation around the axis of direction $\boldsymbol{w}$ by amount of $\theta = ||\boldsymbol{w}||$, using Rodrigues formula (Murray et al. (1994)). To retrieve this representation from a rotation matrix $\boldsymbol{R}$:

$$\theta = arccos(\frac{trace(\boldsymbol{R}) - 1}{2}) \tag{23}$$

and, for $\boldsymbol{R}_{i,j}$ as entries of $\boldsymbol{R}$, we get:

$$\boldsymbol{w} = \frac{1}{2s_\theta} \begin{bmatrix} \boldsymbol{R}_{32} - \boldsymbol{R}_{23} \\ \boldsymbol{R}_{13} - \boldsymbol{R}_{31} \\ \boldsymbol{R}_{21} - \boldsymbol{R}_{12}. \end{bmatrix} \tag{24}$$

As a result, for the rotational part, $\boldsymbol{\Delta}\mathbf{p}_i$ is a vector:

$$\boldsymbol{\Delta}\mathbf{p}_i = [w_{1iT} - w_{1iR}, w_{2iT} - w_{2iR}, w_{3iT} - w_{3iR}]^T \tag{25}$$

Here, the equivalent error parameters are $e_{66} = e_{76}$.

RotZ-error model In this case, parameters that influence only rotation about Z are e_{66} and e_{76} and $\boldsymbol{J}_e$ is a 3 × 2 function of vector $\boldsymbol{e} = [e_{66}, e_{76}]^T$. After analyzing results of compensation with only these errors, it was concluded that they could not fully compensate observed orientational displacements.

RotXYZ- and RotXYZTransXYZ-error model When all rotational errors are considered, parameters that influence only orientation are e_{66}, e_{74}, e_{75}, e_{76} and $\boldsymbol{J}_e$ is a 3 × 4 function of vector $\boldsymbol{e} = [e_{66}, e_{74}, e_{75}, e_{76}]^T$. The parameter evaluation of the selected models on the test set showed the comparably lower value for e_{74} (0.6254). But considering the high values of other parameters (0.9810 and 0.9739), this can be compensated with them.

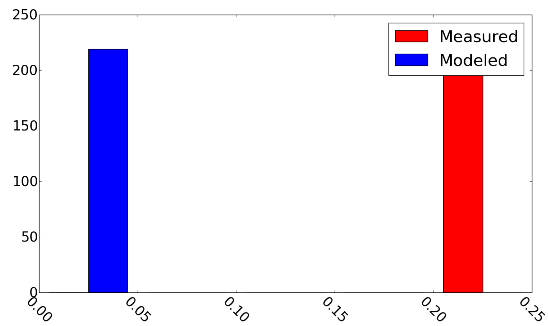

Fig. 5. Orientation error (rad) frequency plot for *RotXYZ*- and *RotXYZTransXYZ*-error model

To illustrate the modeling results the frequency histogram of the absolute difference error between measured and nominal as well as measured and modeled orientation was plotted in Fig. 5, that show significant improvement in orientation accuracy.

Comparing the resulting positional and orientational accuracy of the robot's TCP, one can see that our approach significantly improves both. The statistical results for the test set are presented in table 1 and table 2.

Table 1. Positional errors (mm) before and after modeling

Error	Measured	RotZ	RotXYZ	RotXYZTransXYZ
mean	0.4883	0.2048	0.1870	0.1801
max	1.5895	0.8574	0.5799	0.6893
std	0.2536	0.1167	0.0962	0.0958

Table 2. Orientational errors (rad) before and after modeling

Error	Measured	Modeled
mean	0.2224	0.0061
max	0.2242	0.0080
std	0.0009	0.0007

5. CONCLUSION

We showed a learning based approach for improving accuracy of a KUKA LBR IIWA. We used a generalized error modeling approach based on ridge regression to compensate geometrical and non-geometrical influences without attributing these influences to specific physical processes, but distributing them over introduced error parameters. Three error models were evaluated on the KUKA LBR IIWA. The $RotXYZ$-error and $RotXYZTransXYZ$-error models were able to significantly improve the absolute accuracy of the manipulator's tool position and orientation using a model depending on Cartesian position of the TCP and joint configuration of the robot. What can also be concluded is that the addition of translational errors had no significant impact on the results. It is posiable that the systematic rotation errors due to even smallest manufacturing deviations that could for example lead to non parallel axis are already compensated. To answer this question, we are currently investigating different LBR IIWAs to find out if these rotational errors correlate. Currently, we applied this approach only to positioning errors of the TCP. In an ongoing experiment, we are examining how and if orientation errors of the TCP can be compensated. The compensation of the geometric errors by modifying the kinematic parameters in the robot controller is not always posiable, and for the non-geometric errors the model used in the position control software is even harder to modify. As a result adjusting joint values resulting from the analysis of error models are more practical.

Another important question is to find out, how many training points are needed. We are currently preparing a larger experiment with the goal to find some optimal distribution and minimal number of points necessary for training to reach a given accuracy for a given subregion of the workspace. Finally, we plan to use trajectories instead of points for building the error model. This could reduce training time significantly but requires to take robot dynamics into account.

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